

STATISTICAL ANALYSIS FOR COMPARISON OF OVERALL PERFORMANCE OF PROJECTS USING WEIBULL ANALYSIS ON EARNED VALUE METRICS

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ABSTRACT

Most organizations have evaluated project performance primarily through cost and schedule performance measures, such as earned value management (EVM), for projects. However, to date, few tools in the project management literature have been proposed for enabling comparisons of overall performance of projects. This study introduces a simple index, the CR(t) (critical ratio in time), as a measure of overall performance of a project. The index can easily be calculated on the basis of the earned value data which are normally available from a project cost system. Based on the index, a statistical analysis for comparison of overall performance of projects using Weibull analysis on earned value metrics is proposed. The statistical analysis can be performed on spreadsheet, such as MS EXCEL. Furthermore, the detailed steps in the analysis are discussed along with an example in which five sample projects are analyzed and compared. Based on the obtained results, the author concludes that the proposed approach on the basis of CR(t) data can provide a robust and effective method for managers to evaluate and compare overall performance of projects, and can be applicable in evaluating the project whose value has been determined (e.g., contracted projects).

Keywords: earned value metrics; critical ratio in time; project performance; Weibull analysis

1. INTRODUCTION

The use of project management has a rapid growth in public sectors and private organizations/firms. In general multiple projects are often performed simultaneously in many public organizations and construction companies. Each of these projects is usually a one-time endeavor with a set of well-defined objectives. Projects are managed concurrently and may be either related or independent of one another. At any period in time, projects are either ongoing at various phases, at completion or being terminated for various reasons. To date, the project management literature has contained few tools to enable an effective evaluation and comparison of the overall performance of projects. Moreover, in multi-project organizations it is necessary to develop a simple but reliable method for effectively comparing the performance of projects at a specific time, to help effectively allocate resources, motivate project managers and their teams, and create an improved environment.

For evaluation and comparison purpose, however, a vast amount of various data must be collected due to the multidimensional nature of project performance. Hence, this report introduces a simple in-

dex, called critical-ratio-in-time, as a measure of the overall performance of a project. This index can easily be calculated on the basis of the earned value (EV) data. On the basis of the index, a statistical analysis for comparison of overall performance of projects using Weibull analysis based on earned value metrics is proposed. This statistical analysis can be performed on a spreadsheet, such as MS EXCEL, by using the only information normally available from a project cost system, i.e., to-date actual cost (AC), to-date earned budget, and to-date planned cost. These data are typically collected and calculated on a weekly or other periodic basis for each cost account and summed for the total project.

This paper is organized as follows. Section 2 provides an overview of earned value management. Section 3 relevant literature review on evaluation and comparison of overall performance of projects is given. An overall project performance index proposed is described in section 4; while the Weibull analysis, in section 5. Section 6 presents an illustrated example for analysis. Finally, section 7 concludes the report.

The notations used in this paper are summarized in Table 1.

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2. OVERVIEW OF EARNED VALUE MANAGEMENT

A basic form of earned value analysis can be traced back to industrial engineers on the factory floor in the late 1800s [11]. The use of earned value man-

Table 1. Notation

Symbol	Definition
BAC	Budgeted cost at completion
PV	The planned time-phased baseline of the value of the work scheduled. Also called BCWS—budgeted cost of the work scheduled.
EV	The percent of original budget that has been earned by actual work completed. Also called BCWP—budgeted cost of the work performed.
AC	Actual cost of the work completed. Also called ACWP—actual cost of the work performed.
CV	Cost variance, $CV = EV - AC$
SV	Schedule variance, $SV = EV - PV$
CPI	Cost performance index, a measure of the cost efficiency of the work accomplished to date. $CPI = EV/AC$.
SPI	Schedule performance index (dollar-based), a measure of the schedule efficiency to date. $SPI = EV/PV$.
ES	Earned schedule, a measure identifying the time at which the amount of EV accrued should have been earned.
AT	The elapsed time this has been expended since the start of the project.
SV(t)	Schedule variance in time, $SV(t) = ES - AT$
SPI(t)	Schedule performance index in time, a measure of the schedule efficiency to date. $SPI(t) = ES/AT$.
CR	Critical ratio, a measure of overall project performance, or called cost-schedule index (CSI). The index is the product of CPI and SPI (i.e., $CR = CPI * SPI$)
CR(t)	Critical ratio in time, a revised measure of overall project performance. The index is the product of CPI and SPI (i.e., $CR(t) = CPI * SPI(t)$)
X_cum (i)	Cumulative value of X index at time period <i>i</i> .
X_per (i)	Periodic value of X index at time period <i>i</i> .

agement (EVM) as a project monitoring and control mechanism began in the 1960s, when it was championed by the U.S. federal government as an integral part of the cost/schedule control systems criteria (C/SCSC) for its agencies and contractors to track project performance [30, p. 112]. EVM has proven to be such a useful tool that it has been approved as an ANSI/EIA 748-A Standard [23] and has been used extensively by the private sector, local governments and the federal government for management of various types of projects— infrastructure, information technology, manufacturing, etc [2]). Daniel [8] even pointed out that “If an organization can effectively integrate EVM into their procurement, timekeeping, and executive information systems, then EVM is probably the best single method for measuring and reporting true project performance and estimating time and cost to complete.”

Project Management Institute (PMI) defines EVM as a management methodology for integrating the project’s scope, schedule, and resources, and for objectively measuring project performance and progress from project initiation through closeout [24]. This is accomplished by calculating a number of measures. First, the budgeted cost for work scheduled (BCWS) within a given time period, also called planned value (PV), is calculated. Second, the actual

cost of work performed (ACWP) during the given time period, also called actual cost (AC), and the budgeted cost for work performed (BCWP), also called earned value (EV), are calculated.

There are two well-known performance indices for evaluation of project performance in the EVM [12]. One is called cost performance index (CPI) which is a measure of the cost efficiency of the work accomplished to date. The other is called schedule performance index (SPI) which is a measure of the schedule efficiency to date.

The performance indices are determined next to show the percentage of variation, between planned and actual performance. The general expressions for the cumulative cost- and schedule- variance as well as cost- and schedule- performance indexes are:

$$\text{Schedule variance (SV)} = EV - PV \quad (1)$$

$$\text{Cost variance (CV)} = EV - AC \quad (2)$$

$$\text{Schedule performance index (SPI)} = EV/PV \quad (3)$$

$$\text{Cost performance index (CPI)} = EV/AC \quad (4)$$

The above-mentioned formulas are used to calculate the performance indices, generally based on cumulative data. If CPI/SPI is less than 1 (equal to 1, larger than 1), the cost/schedule efficiency is lower than (equal to, higher than) planned. Both sets of indicators are computed at specific times, usually

monthly. The CPI is the most accepted and used index. It has been tested over time and found to be the most accurate, reliable, and stable [12, p.435]. However the

SPI behaves erratically for projects behind schedule and loses predictive ability over the last third of the

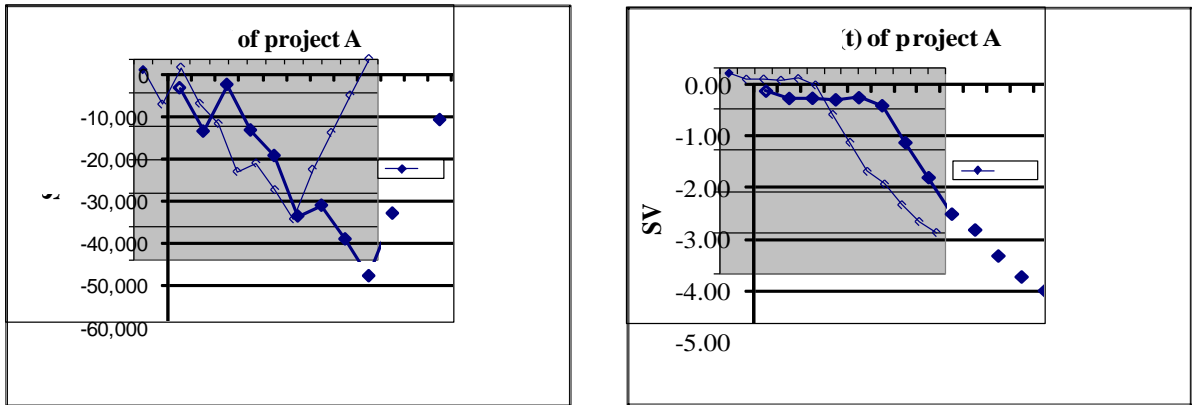


Figure 1. SV vs. SV(t) of project A

project, since it uses monetary terms as an analogue of time, which is not always correct [17]. At the end of a project, the $EV = PV = BAC$ (budget at completion), and hence, the SPI always equals to unity (i.e., 1) [17, 29].

Lipke [17] provides a time-based measure, earned schedule (ES), to overcome the quirky behavior of the SPI index, and calculates alternative schedule performance measure (referred to as SPI(t)) that are directly expressed in time units. ES is the point in time when the current EV was to be accomplished [27]. This novel method relies on similar principles of the earned value method, and can be calculated as follows [28]: Find period n such that $EV \geq PV_n$ and $EV < PV_{n+1}$

$$ES = n + (EV - PV_n) / (PV_{n+1} - PV_n) \quad (5)$$

n represents the number of the time period from the beginning of the project.

That is, ES is the number of completed PV time increments EV exceeds plus the fraction of the incomplete PV increment. With ES determined, time-based indicators can be formed. The corresponding cumulative schedule performance index is:

$$SV(t) = ES - AT \quad (6)$$

$$SPI(t) = ES / AT \quad (7)$$

where AT is used to refer to the elapsed time which has been expended since the start of the project.

Using ES, schedule indicators can be formed which behave appropriately and analogously to the cost indicators [13, 17]. The Schedule Variance, $SV(t)$, is positive when the ES exceeds AT, and, certainly, is negative when it lags. The Schedule Performance Index, $SPI(t)$, is greater than 1.0 when ES is larger than AT, and, certainly, is less than 1.0, when ES is less than AT.

Here sample data of Re-vamp check-in project (referred as project A), shown in Table 3, from Vandevoorde and Vanhoucke [29], is provided and used to explain the above-mentioned. The duration of project A, with a budget at completion of € 360.738, is 9 months. The project was delivered 4 months later than

expected, but under budget. The graph of the SV along the project duration shows that the SV follows a negative trend till ninth month, followed by a positive trend and finally ending with a zero variation (the left graph of Figure 1). The graph of the $SV(t)$, on the contrary, shows a negative trend along the complete project duration, and ends with a cumulative variation of -4 months, which is exactly the project's delay (the right graph of Figure 1). A similar effect is revealed in the graph of the schedule performance metrics, shown Figure 2. During the early and middle stages, both SPI and $SPI(t)$ correlate very well. However, towards the late project stage (at the around 75% completion point), the SPI becomes unreliable showing an improving trend while the project is slipping further away. This further performance decline is clearly shown by the $SPI(t)$ indicator.

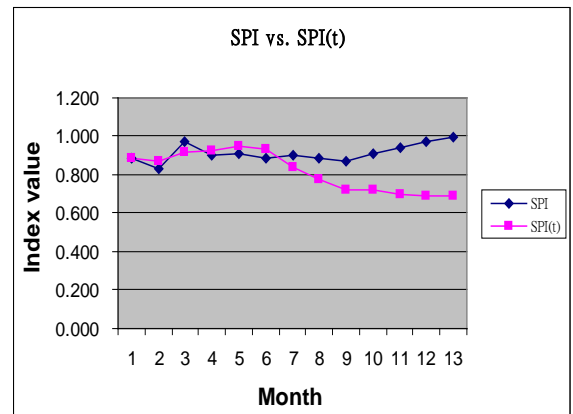


Figure 2. SPI vs. SPI(t) of project A

3. RELEVANT LITERATURE REVIEW

In today's competing business environment, an important issue in a multi-project organizations is looking for a method for effectively evaluating and comparing the performance of various projects at a given time period. The method is necessary that it can help senior management effectively allocate resources,

motivate project managers and their teams and create an improvement environment. However, few works on this issue are reviewed as follows.

Rozenes *et al.* [26] developed an alternative approach called the multidimensional project control system (MPCS), which provides a control mechanism, for monitoring the characteristics of a project (e.g., quality, design, functionality, and operations). Ten dimensions of project performance measures were identified by the Project Management Institute (PMI) [25] for studying benchmarking efforts (e.g., cost, schedule performance, staffing, alignment to strategic business goals, and customer satisfaction). Despite its multidimensional nature, most organizations have traditionally evaluated project performance primarily through cost and schedule performance measures, such as EVM [10].

Vitner *et al.* [31] investigated the possibility of using the data envelope analysis (DEA) approach for evaluating the performance of projects in a multi-project environment. Each project is viewed as a decision-making unit (DMU) having its own inputs and outputs. The efficiency of a project would then be a weighted sum of its outputs divided by a weighted sum of its inputs. This research team demonstrates that DEA can be successfully applied to the evaluation of multi-project environment. The approach used in the illustration is an integration of the MPCS and EVM methods, but it is clear that either method could have also been used separately.

Farris *et al.* [10] also presented a case study to show how DEA was applied to generate objective cross-project comparisons of project duration within an engineering department of the Belgian Armed Forces. They applied DEA to demonstrate that “Does the new concurrent engineering design process appear to result in shorter project duration than the old serial engineering design process, given differences in characteristics across projects?” When applying DEA to compare the relative efficiency of projects, the most concern considered is the size of the comparison group. A rule of thumb generally adopted in DEA applications is that the number of DMUs should be at least twice the sum of the number of inputs and outputs. However, in multi-project environments, it is very common that the number of projects (i.e., the DMUs) may be relatively small and hence the DEA rule of thumb may not be achieved. Thus, there is a need for a methodology to reduce the inputs and outputs to meet the rule of thumb. This is a major drawback.

From the above literature review, for evaluation and comparison purpose a vast amount of various data must be collected due to the multidimensional nature of project performance. In a performance evaluation framework where senior management wishes to minimize the number of performance measures it employs, while ensuring maximal coverage or visibility

into the project, having a tool that captures each of the three areas: scope, schedule, and budget, would be ideal [2]. Therefore, this report attempts to propose a simple index as a measure of the overall performance of a project. The index not only can easily be calculated on the basis of the earned value data which are normally available from a project cost system, but meets senior management wishes.

4. OVERALL PROJECT PERFORMANCE INDEX

Although SPI and CPI are useful for project managers to evaluate and monitor the cost and schedule performance of a project, either of the indices does not reflect the overall performance of a project. Moreover, it is possible for one of the indexes (e.g. CPI) to be favorable while the other (e.g. SPI) is unfavorable. An ongoing/completed project might be behind/ ahead of/on schedule but under/at/over cost. There are nine possibilities in total. When comparison of overall performance of projects in an organization is desired, a composite index is necessitated. For example, overall performance of the following two ongoing projects is being compared. Project X is behind schedule (assumed SPI = 0.9) but under cost (CPI = 1.2); while project Y is ahead of schedule (SPI = 1.1) but with a cost over-run (CPI = 0.9). If considering comparison of a single cost/schedule performance of projects X and Y, one may easily conclude that project Y is superior to project X in terms of schedule performance; while Y is inferior to X in terms of cost performance. Based on SPI/CPI, it is very difficult to evaluate and differentiate which one has better overall performance. This leads to why we need a simple and effective index as a measure of the overall performance of a project.

Many works [3, 4, 7, 16, 21] advocated that critical ratio (CR) or called cost-schedule index (CSI), $CR = CPI * SPI$, is usually used as a measure of overall project performance. A CR of 1.00 indicates that the overall project performance is on target. A CR of more than (or less than) 1.00 indicates that the overall project performance is excellent (or poor). Cable *et al.* [5] also pointed out that the CPI and SPI are only individual index of each project, whereas the Critical Ratio (CR) considering both the CPI and SPI reflects the overall status of the projects.

But in practice, the CR may not be an inappropriate one to measure the overall project performance owing to the SPI index with some anomalies discussed in section 2. Hence, a revised index, called critical ratio in time, $CR(t) = CPI * SPI(t)$, is presented and used to measure the overall project performance in this paper. Similar to CR, a $CR(t)$ of 1.00 indicates that the overall project performance is on target. This might result from both CPI and $SPI(t)$ being close to target, or, if one of these indices suggests poor performance, the other must be indicating good performance. This

allows some trade-offs to reach the desired project goals. A CR(t) of more than (or less than) 1.00 indicates that the overall project performance is excellent (or poor).

In the following the author wants to address and present the applicability of Weibull analysis for evaluating CR(t) and to provide managers with an effective tool for stochastically evaluating the CR(t). The reasons to use Weibull analysis are addressed as follows. Firstly, a number of probabilistic distributions such as exponential, logistic, normal, lognormal, and Weibull, are checked against the data using Anderson-Darling test (using *Easy-Fit* data analysis tool) and the Weibull distribution is found to be the best representative for CR(t) (having the lowest AD value). Secondly, since the Weibull distribution is robust enough to assume a number of different distributions (including the normal, exponential and beta distributions) one can employ the Weibull distribution to model the CR(t) data of the project, without the limitations of a predefined distribution assumption [22]. Thirdly, Weibull analysis can provide accurate performance analysis and risk predictions with extremely small samples [1]. Small sample can be defined as any sample less than 25.

To utilize Weibull analysis for evaluating and comparing CR(t), the periodic values of CPI, SPI(t) and CR(t) is necessitated and can be derived from cumulative values. Those values are computed from the differences in their respective cumulative values for successive periods. Thus, the periodic formulas for CPI_per(i), SPI(t)_per(i), and CR(t)_per(i) are as follows.

$$\text{CPI_per}(i) = \frac{(\text{EV_cum}(i) - \text{EV_cum}(i-1))}{(\text{AC_cum}(i) - \text{AC_cum}(i-1))} \quad (8)$$

$$\text{SPI}(t)_\text{per}(i) = \frac{(\text{ES_cum}(i) - \text{ES_cum}(i-1))}{(\text{AT_cum}(i) - \text{AT_cum}(i-1))} \quad (9)$$

$$\text{CR}(t)_\text{per}(i) = \text{CPI_per}(i) \times \text{SPI}(t)_\text{per}(i) \quad (10)$$

where EV_cum(i), AC_cum(i), and AT_cum(i) denote the respective cumulative values from the beginning of the project to period *i* for EV, AC, and AT.

5. WEIBULL ANALYSIS

Used to model data sets containing values greater than zero, such as failure data, Weibull analysis can perform several functions such as making predictions about a product's life and comparing the reliability of competing product designs [9]. The statistical method for plotting and evaluating data using Weibull analysis was developed by Johnson [14, 15]. It is assumed that a straight line is representative of the data, although in many cases the data loci are not linear. By using the method of least squares, a straight line is drawn through an array of points on each plot. Dorner [9] originally presented an example showing a detailed procedure for using Microsoft EXCEL to perform Weibull analysis, which was used to compare the reliability of two proposed designs for a jack-in-the-box spring housing in a toy-manufacturing

company. Nassar *et al.* [22] presented a similar approach for evaluating the schedule performance of two ongoing projects.

Weibull analysis essentially entails fitting a Weibull distribution to a collected data set about some characteristic of a system (usually a quality or performance characteristic), and assessing the reliability of the system on the basis of the fitted distribution [22]. The Weibull probability density function (PDF) is given by [20]:

$$f(x) = \frac{\beta}{\alpha} \left(\frac{x-\gamma}{\alpha} \right)^{\beta-1} \exp \left[- \left(\frac{x-\gamma}{\alpha} \right)^{\beta} \right], \quad (11)$$

$$\beta > 0, \alpha > 0, x > \gamma > 0$$

The Weibull cumulative distribution function (CDF) is given by:

$$F(x) = 1 - \exp \left[- \left(\frac{x-\gamma}{\alpha} \right)^{\beta} \right] \quad (12)$$

Parameters α , β , and γ are the shape, scale, and location parameters, respectively. From eq. (12) and letting $\gamma = 0$, then make the double logarithmic transformation of the CDF. The relation between CDF and two parameters (α , β) can be obtained as follows.

$$\ln \ln \left[\frac{1}{1-F(x)} \right] = \beta \ln x - \beta \ln \alpha \quad (13)$$

Equation (13) is an equation of a straight line. Comparing this equation to a straight line: $y = mx + b$. We see that the left side of the equation corresponds to y , $\ln x$ corresponds to x , β corresponds to m , and $-\beta \ln \alpha$ corresponds to b . Thus, when we perform the linear regression, the estimate for the Weibull β parameter comes directly from the slope of the line. The estimate for α parameter must be calculated as follows:

$$\alpha = e^{-\left(\frac{b}{\beta}\right)}.$$

Many methods (such as median rank method, least squares, and maximum likelihood) exist for estimating the Weibull distribution parameters (α , β) from a given data set. The one used here is the median ranking method. The median rank of each data point is calculated next as $(\text{rank no.} - 0.3) / (\text{no. of points} + 0.4)$. The main advantage of the median rank method is its relative simplicity and ease of use, which makes it an ideal method for project managers. The underlying concept of this method is that, by using an appropriate transformation, the two-parameter Weibull model (α , β) can be represented by a straight line; therefore, the two parameters (α , β) can be determined by using simple linear regression [19].

Steps in Weibull analysis

In this research, Weibull analysis is presented and used in conjunction with EVM, a periodic (not cumulative) overall performance index. Similar to the

procedure used by Dorner [9] and Nassar *et al.* [22], a series of steps for performing Weibull analysis on spreadsheets is summarized as follows.

Step 1. Collect the PV, EV, and AC data on the first project and calculate the periodic values of the ES, CPI, SPI(t) and CR(t);

Step 2. Rank the periodic values of the CR(t) in ascending order (the smallest value is assigned a rank of 1);

Step 3. Calculate the median rank (rank No. - 0.3) / (No. of points + 0.4);

Table 2. Scenarios of projects A to E

Project	BAC	AC	EV	Planned duration	Actual duration	Status
A	360,738	349,379	360,738	9	13	completed
B	2,875	3,247	2,875	9	12	completed
C	906	932	906	10	9	completed
D	91,000	94,126	84,360	19	-	ongoing
E	91,000	106,500	85,995	24	-	ongoing

Note: Projects A to E are expressed in thousands of €

Table 3. Periodic values of project A

Month (i)/ AT	PV	EV	AC	ES_cum	ES_per(i)	SPI(t)_per(i)	CPI_per(i)	CR(t)_per(i)
1	28,975	25,645	25,567	0.885	0.885	0.885	1.003	0.888
2	81,681	68,074	66,293	1.742	0.857	0.857	1.042	0.893
3	91,681	89,135	78,293	2.745	1.004	1.004	1.755	1.762
4	138,586	125,244	124,073	3.716	0.970	0.970	0.789	0.765
5	218,141	198,754	191,367	4.756	1.041	1.041	1.092	1.137
6	302,478	268,763	259,845	5.600	0.844	0.844	1.022	0.863
7	323,632	292,469	285,612	5.881	0.281	0.281	0.920	0.259
8	345,876	306,725	290,843	6.201	0.319	0.319	2.725	0.871
9	360,738	312,864	303,489	6.491	0.290	0.290	0.485	0.141
10		327,694	316,431	7.183	0.692	0.692	1.146	0.793
11		338,672	320,690	7.676	0.494	0.494	2.578	1.272
12		349,861	336,756	8.268	0.592	0.592	0.696	0.412
13		360,738	349,379	9.000	0.732	0.732	0.862	0.631

Table 4. Periodic values of project B

Month(i)/ AT	PV	EV	AC	ES_cum	ES_per(i)	SPI(t)_per(i)	CPI_per(i)	CR(t)_per(i)
1	375	325	344	0.867	0.867	0.867	0.945	0.819
2	525	427	452	1.347	0.480	0.480	0.944	0.453
3	850	735	796	2.646	1.299	1.299	0.895	1.163
4	1,355	1,025	1,056	3.347	0.700	0.700	1.115	0.781
5	1,768	1,453	1,562	4.237	0.891	0.891	0.846	0.753
6	2,125	1,774	1,922	5.017	0.780	0.780	0.892	0.695
7	2,452	2,024	2,256	5.717	0.700	0.700	0.749	0.524
8	2,625	2,190	2,451	6.199	0.482	0.482	0.851	0.410
9	2,875	2,356	2,676	6.706	0.508	0.508	0.738	0.375
10		2,565	2,925	7.653	0.947	0.947	0.839	0.795
11		2,735	3,138	8.440	0.787	0.787	0.798	0.628
12		2,875	3,247	9.000	0.560	0.560	1.284	0.719

Table 5. Periodic values of project C

Month (i)/ AT	PV	EV	AC	ES_cum	ES_ per(i)	SPI(t)_ per(i)	CPI_ per(i)	CR(t)_ per(i)
1	34	36	35	1.059	1.059	1.059	1.029	1.089
2	87	93	95	2.086	1.027	1.027	0.950	0.976
3	157	169	174	3.056	0.970	0.970	0.962	0.933
4	373	402	412	4.165	1.109	1.109	0.979	1.086
5	549	597	623	5.387	1.222	1.222	0.924	1.130
6	673	735	754	6.496	1.109	1.109	1.053	1.168
7	798	839	874	7.932	1.436	1.436	0.867	1.244
8	842	887	932	8.367	0.435	0.435	0.828	0.360
9	876	906	952	10.000	1.633	1.633	0.950	1.552
10	906							

Table 6. Periodic values of project D

Month (i)/ AT	PV	EV	AC	ES_cum	ES_ per(i)	SPI(t)_ per(i)	CPI_ per(i)	CR(t)_ per(i)
1	500	910	1,166	1.820	1.820	1.820	0.780	1.420
2	3500	3640	4,136	2.028	0.208	0.208	0.919	0.191
3	8500	10010	9,763	3.137	1.109	1.109	1.132	1.256
4	14500	13650	15,192	3.858	0.721	0.721	0.670	0.483
5	18000	18200	19,468	5.040	1.182	1.182	1.064	1.257
6	23000	25480	25,883	6.496	1.456	1.456	1.135	1.652
7	28000	30940	32,299	7.980	1.484	1.484	0.851	1.263
8	31000	35490	37,311	9.198	1.218	1.218	0.908	1.106
9	34500	38220	41,708	9.744	0.546	0.546	0.621	0.339
10	39500	40040	46,642	10.098	0.354	0.354	0.369	0.131
11	45000	43680	52,276	10.760	0.662	0.662	0.646	0.428
12	52000	50960	61,498	11.851	1.091	1.091	0.789	0.862
13	59500	62790	68,015	13.658	1.807	1.807	1.815	3.279
14	64500	67340	72,765	14.516	0.858	0.858	0.958	0.822
15	70000	72800	76,804	15.560	1.044	1.044	1.352	1.411
16	75000	76440	79,806	16.240	0.680	0.680	1.213	0.825
17	81000	80080	87,170	16.847	0.607	0.607	0.494	0.300
18	86500	83720	91,399	17.495	0.648	0.648	0.861	0.558
19	91000	84360	94,126	17.611	0.116	0.116	0.235	0.027

Step 4. Calculate the $\ln\{\ln[1/(1-\text{median rank})]\}$ values and draw a straight line (trend line in MS EXCEL) between them and the $\ln \text{CR}(t)$ in separate charts; and choose the option of showing the straight line equation (in the form of $y = mx + b$);

Step 5. Set $\beta = m$ and $\alpha = e^{b/\beta}$;

Step 6. Add the $\text{CR}(t)$ index in increments of 0.1 from 0.0 to 2.0 and use Weibull function=WEIBULL (index value, β , α , TRUE) in MS EXCEL to calculate the performance probability for the Weibull distribution.

Step 7. Draw the PDF against the $\text{CR}(t)$ index value.

Table 7. Periodic values of project E

Month (i)/ AT	PV	EV	AC	ES_cum	ES_ per(i)	SPI(t)_ per(i)	CPI_ per(i)	CR(t)_ per(i)
1	1,500	455	1,000	0.303	0.303	0.303	0.455	0.138
2	4,000	1820	2,500	1.128	0.825	0.825	0.910	0.750
3	8,000	3640	5,000	1.856	0.728	0.728	0.728	0.530
4	12,000	6370	8,000	2.593	0.737	0.737	0.910	0.670
5	16,000	9100	12,000	3.275	0.683	0.683	0.683	0.466
6	19,500	11830	16,000	3.958	0.683	0.683	0.683	0.466
7	23,000	14560	19,500	4.640	0.683	0.683	0.780	0.532
8	26,500	18200	23,000	5.629	0.989	0.989	1.040	1.028
9	30,000	22750	30,500	6.929	1.300	1.300	0.607	0.789
10	34,500	25480	35,000	7.709	0.780	0.780	0.607	0.473
11	38,000	28210	41,000	8.489	0.780	0.780	0.455	0.355
12	43,000	32760	48,000	9.613	1.125	1.125	0.650	0.731
13	48,000	38220	54,000	11.044	1.431	1.431	0.910	1.302
14	51,500	40950	60,000	11.590	0.546	0.546	0.455	0.248
15	58,000	48230	67,000	13.066	1.476	1.476	1.040	1.535
16	62,000	52780	73,000	14.197	1.131	1.131	0.758	0.858
17	66,000	57330	77,000	14.897	0.700	0.700	1.138	0.796
18	70,000	63700	81,500	16.425	1.528	1.528	1.416	2.163
19	76,000	69160	87,000	17.790	1.365	1.365	0.993	1.355
20	79,500	73710	92,000	18.618	0.828	0.828	0.910	0.754
21	84,000	77350	96,000	19.386	0.767	0.767	0.910	0.698
22	87,000	80990	100,000	20.331	0.945	0.945	0.910	0.860
23	90,000	83720	102,500	20.938	0.607	0.607	1.092	0.662
24	91,000	85995	106,500	21.665	0.727	0.727	0.569	0.414

Table 8. Median ranks method of fitting a data set to a Weibull distribution

CR(t), Project A	Rank	Median Ranks	1/(1-median rank)	$\ln\{\ln[1/(1-\text{median rank})]\}$	$\ln(\text{Project A, CR(t)})$
0.141	1	0.052238806	1.05511811	-2.925223234	-1.959846248
0.259	2	0.126865672	1.145299145	-1.99756029	-1.35245892
0.412	3	0.201492537	1.252336449	-1.491606142	-0.886029301
0.631	4	0.276119403	1.381443299	-1.129704207	-0.46102723
0.765	5	0.350746269	1.540229885	-0.839487848	-0.267606957
0.793	6	0.425373134	1.74025974	-0.59052854	-0.232515274
0.863	7	0.5	2	-0.366512921	-0.147577036
0.871	8	0.574626866	2.350877193	-0.156901171	-0.138596488
0.888	9	0.649253731	2.85106383	0.046589839	-0.119038605
0.893	10	0.723880597	3.621621622	0.252253233	-0.11363357
1.137	11	0.798507463	4.962962963	0.47125468	0.128296057
1.272	12	0.873134328	7.882352941	0.724949317	0.24067984
1.761	13	0.947761194	19.14285714	1.082459075	0.56607794

6. EXAMPLE

In this section, a detailed description of Weibull analysis by using earned value metrics is illustrated in five projects, each of projects with different size ranging from 9 to 24 and different performance, is presented. The data of the five construction projects are summarized in Table 2. Projects A, B and C, drawn from Vandevorode and Vanhoucke [28], are already completed, while projects D and E, from Nassar *et al.* [22], are still on-going. The duration of project A is 9 months, with a budget at completion of €360,738. At completion, project A is delivered 4 months later than expected, but under budget with actual cost €349,379. Completed project B is behind schedule with a cost-run, while completed project C, ahead of schedule but under cost. Projects D and E are ongoing but approaching completion both are lag behind and cost over-run. The two projects have similar schedule performance, but probably different cost performance. Before proceeding the Weibull analysis, the periodic values of the ES, CPI, SPI(t) and CR(t) metrics must be calculated based on the PV, EV, and AC data of each project. The detailed procedures of computing the foregoing metrics are introduced in section 4, and the computational results are shown in Tables 3-7.

F test of the analysis of variance reveals no statistical difference among average CR(t) for the five projects (*p*-value = 0.42) on the data in Tables 3-7. Although the sample average is a simple measure of central tendency, it gives no information about the spread or shape of the distribution of overall performance index. Could the average values of CR(t) for the five projects be not significantly different, however, the overall project performance associated with each is quite different? How can one be more scientific

about comparing the overall project performance associated with the five projects?

6.1 Preparing to analyze

Modeling the data of an overall performance index using Weibull analysis requires some preparation. By using the data of project A (shown in Table 3), an example of the ES computation can be illustrated. The EV is accrued at the end of month 3, thereby making AT equal to 3. Thus, EV = 89,135; moreover, all of the PV through month 2, PV₂ = 81,681, has been earned. However, only a portion of month 3 has been completed. Therefore, the duration of the completed portion of the planned schedule is in excess of 2 months; thus, *n* = 2. ES_{cum}(3) = *n* + (EV - PV_{*n*}) / (PV_{*n+1*} - PV_{*n*}) = 2 + (89,135 - 81,681) / (91,681 - 81,681) = 2.745. To calculate the overall performance index CR(t) at the end of month 3, the respective periodic values for ES, CPI, and SPI(t) must be computed in advance. The computational details are described as follows: ES_{cum}(2) = 1 + (EV - PV₁) / (PV₂ - PV₁) = 1 + (68,074 - 28,975) / (81,681 - 28,975) = 1.742. The respective periodic values for month 3 are computed next: SPI(t)_{per}(3) = (ES_{cum}(3) - ES_{cum}(2)) / (AT_{cum}(3) - AT_{cum}(2)) = (2.745 - 1.742) / (3 - 2) = 1.003. Then, CPI_{per}(3) = (EV_{cum}(3) - EV_{cum}(2)) / (AC_{cum}(3) - AC_{cum}(2)) = (89,135 - 68,074) / (78,293 - 66,293) = 1.755. Hence, CR(t)_{per}(3) = SPI(t)_{per}(3) × CPI_{per}(3) = 1.004 × 1.755 = 1.762. The calculated periodic values for project A are given in Table 3. Similarly, the values for projects B to E are listed in Tables 4-7, respectively.

6.2 Fitting a line to data

Once the periodic values for CR(t) of a project have been calculated, the next step is to fit the CR(t) data of each project to the Weibull CDF by proceeding

to Steps 2-4, shown in Table 8. A simple approach, the median ranking method, is used to estimate the distribution of the parameters (α , β). The approach is first to rank the CR(t) values for project A in ascending order and place their respective rank in the second column. The median rank of each data point is calculated next as $(\text{rank no.} - 0.3) / (\text{no. of points} + 0.4)$. For example, for project A the third point median rank is equal to $(3 - 0.3) / (13 + 0.4) = 0.2015$. As shown in Equation (13), values of $\ln\{\ln[1/(1-\text{median rank})]\}$ and $\ln(\text{CR}(t))$ for the CR(t) data points are calculated in columns 5-6. At this point, the Weibull analysis is ready to be performed. The Analysis ToolPak add-in that is built into MS EXCEL was used for the regression analysis to evaluate scale and shape (α and β) parameters. It finds that $\beta = 1.629$ and $\alpha = 0.962$. An identical analysis using the projects B to E data yields the respective values of β and α . Table 9 lists the parameters for each project from regression analysis.

Table 9. The parameters for each project

Project	Intercept	$\ln(\text{Project}^*, \text{CR}(t))$	p-value	Scale parameter (α)	Shape parameter (β)
Project A	0.0622	1.6294	0.0000	0.9625	1.6294
Project B	0.9339	3.3278	0.0000	0.7553	3.3278
Project C	-0.5155	2.3971	0.0013	1.2399	2.3971
Project D	-0.0449	1.0406	0.0000	1.0441	1.0406
Project E	0.2598	1.9676	0.0000	0.8763	1.9676

6.3 Interpreting the results

The Weibull shape parameter, called β , indicates whether the failure rate is increasing, constant or decreasing. Here, the shape parameter β indicates whether the overall performance of the project in terms of CR(t) is increasing, constant or decreasing. A $\beta > 1.0$ indicates that the project has an increasing overall performance, i.e., there is an improvement in the performance of the project from one index value to another. A $1 < \beta < 2$ indicates that the overall performance increases at a decreasing rate as index value increases. When $\beta > 2$, the overall performance is on a slope, increasing as the index value increases. A $\beta = 1.0$ indicates that the project has a constant overall performance index rate; however, a $\beta < 1.0$ indicates a decreasing index rate. The desired rate is therefore a $\beta \geq 1$. In the example, the shape parameter values of projects A, D and E are 1.6294, 1.0406, and 1.9676, respectively, indicating that the overall performance of this project increases at a decreasing rate as the index value increases. In addition, the shape parameter values of projects B and C are 3.3278 and 2.3971, respectively, both of which are larger than 2.0. These values indicate that the overall performance of projects B and C is on a slope, thus increasing as the index value increases. From this perspective, projects B and C is superior to that of the others.

The scale parameter α (the Weibull characteristic life) is a measure of performance variability. A high α indicates more variability in the overall project performance in terms of the index values. The Weibull characteristic life α is a measure of the scale, or spread, in the distribution of data, which happens to equal the value at which 63.2 percent of the overall performance index has failed to achieve. In the example, project C and D have about 37 percent of the overall performance index, which succeeds in attaining 1.24 and 1.044, respectively; whereas projects A, E, and B, achieving 0.962, 0.8763 and 0.755, respectively. From this perspective, project C is the best; the next, project D; and the worst, Project B.

In the foregoing discussion, in terms of overall performance, it is difficult to determine which project is the best. Hence, the reliability of the overall performance of each project is calculated next. Calculating reliability of overall performance

The next step is to determine the reliability of achieving a particular index value for overall performance, i.e., the CR(t). If the probability of achieving a CR(t) value close-to or more-than 1 is high, then this indicates that the overall project performance has a strong chance to finish on target. This can be thought of as the reliability of achieving a particular CR(t) value and is equal to 1-“the performance probability.” The performance probability can be easily calculated by using the built-in Weibull function in EXCEL as:=WEIBULL (index value, shape parameter, scale parameter, TRUE). TRUE indicates a cumulative distribution function. The range of index values of overall performance is set at 0.1 to 2.0 in 0.1 increments. Table 10 shows the performance probability and the reliability of each project.

6.4 Comparing of overall performance

In the final step, the five projects are compared in terms of reliability by using a performance graph, the results of which are plotted in Figure 3, which allows a comprehensive comparison of performance probability. A number of issues can be pointed out from the graph (or one can examine the probability of an index values). From performance graph one can examine the probability of achieving a particular CR(t) value for a project. Thus, the overall performance of each project can be easily evaluated and compared. In the example, one can see that project C has about 0.554 probability

Table 10. Reliability of each project

Index value	Project A		Project B		Project C		Project D		Project E	
	Performance Probability	Reliability	Performance Probability	Reliability	Performance Probability	Reliability	Performance Probability	Reliability	Performance Probability	Reliability
0.1	0.0247	0.9753	0.0012	0.9988	0.0024	0.9976	0.0834	0.9166	0.0139	0.9861
0.2	0.0744	0.9256	0.0119	0.9881	0.0125	0.9875	0.1640	0.8360	0.0532	0.9468
0.3	0.1390	0.8610	0.0452	0.9548	0.0328	0.9672	0.2390	0.7610	0.1143	0.8857
0.4	0.2127	0.7873	0.1136	0.8864	0.0643	0.9357	0.3082	0.6918	0.1924	0.8076
0.5	0.2910	0.7090	0.2238	0.7762	0.1072	0.8928	0.3717	0.6283	0.2822	0.7178
0.6	0.3706	0.6294	0.3718	0.6282	0.1610	0.8390	0.4299	0.5701	0.3779	0.6221
0.7	0.4485	0.5515	0.5399	0.4601	0.2243	0.7757	0.4830	0.5170	0.4742	0.5258
0.8	0.5228	0.4772	0.7020	0.2980	0.2952	0.7048	0.5314	0.4686	0.5665	0.4335
0.9	0.5919	0.4081	0.8333	0.1667	0.3712	0.6288	0.5755	0.4245	0.6514	0.3486
1.0	0.6550	0.3450	0.9215	0.0785	0.4496	0.5504	0.6156	0.3844	0.7266	0.2734
1.1	0.7115	0.2885	0.9696	0.0304	0.5279	0.4721	0.6521	0.3479	0.7907	0.2093
1.2	0.7612	0.2388	0.9906	0.0094	0.6033	0.3967	0.6852	0.3148	0.8437	0.1563
1.3	0.8044	0.1956	0.9977	0.0023	0.6737	0.3263	0.7153	0.2847	0.8861	0.1139
1.4	0.8414	0.1586	0.9996	0.0004	0.7376	0.2624	0.7426	0.2574	0.9190	0.0810
1.5	0.8726	0.1274	0.9999	0.0001	0.7937	0.2063	0.7673	0.2327	0.9438	0.0562
1.6	0.8986	0.1014	1.0000	0.0000	0.8416	0.1584	0.7897	0.2103	0.9620	0.0380
1.7	0.9201	0.0799	1.0000	0.0000	0.8812	0.1188	0.8100	0.1900	0.9749	0.0251
1.8	0.9375	0.0625	1.0000	0.0000	0.9131	0.0869	0.8284	0.1716	0.9838	0.0162
1.9	0.9516	0.0484	1.0000	0.0000	0.9381	0.0619	0.8450	0.1550	0.9898	0.0102
2.0	0.9628	0.0372	1.0000	0.0000	0.9570	0.0430	0.8601	0.1399	0.9937	0.0063

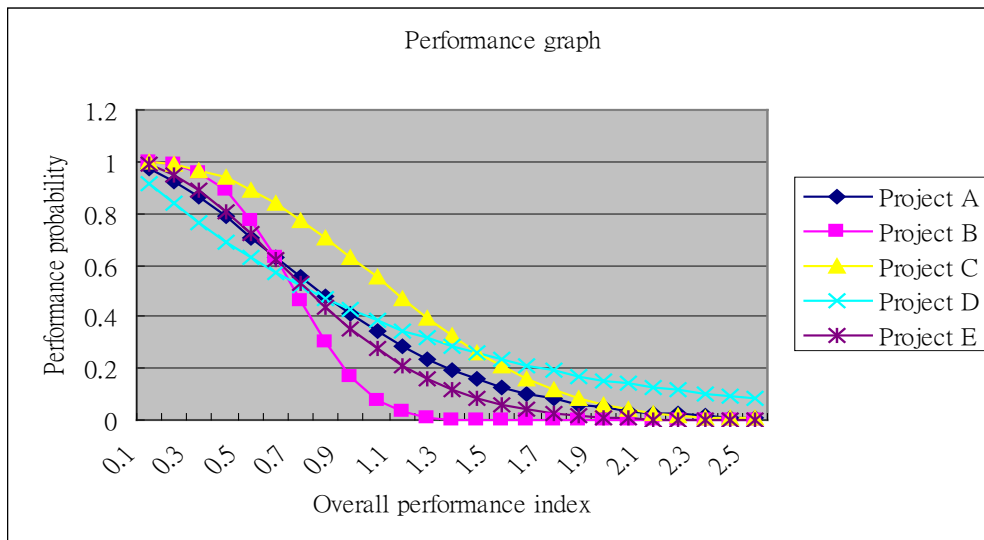


Figure 3. Performance graph

of achieving the target, $CR(t) = 1.0$; whereas projects D, A, E, and B only about 0.384, 0.345, 0.273, and 0.079, respectively.

According to Chang [6] index values equal to or above 0.9 indicate average to above-average performance (from average to excellent) as well as index values less than 0.9 indicate performance less than average (from average to unsatisfactory). From Table 10 or Figure 3, project C always has the highest probability of attaining the above-average performance, for $CR(t) = 0.9$ to 1.4; project D, the second; and then, projects A, E, and B, in sequence. Hence, one can conclude that project C has the best overall performance; project D, the second; then, A, E; whereas project B, the worst.

Based on the obtained results of Weibull analysis, the proposed approach can help management effectively evaluate and compare the overall performance of projects, though there may be no significantly difference among projects by F-test.

7. CONCLUSION

In many organizations, more than one project is often executed concurrently. A major issue for each organization is how to measure and evaluate the performance of each project and to compare the overall performance of the various projects. This report has first introduced a novel overall project performance index and proposed a simple and reliable approach which applies Weibull analysis to evaluate and compare the relative overall performance on earned value metrics. The index can easily be calculated on the basis of the earned value data which are normally available from a project cost system instead of collecting a vast amount of complex data. An example consisting of five actual projects has illustrated a step-by-step approach for evaluating and comparing overall project performance. Moreover, the analysis is fairly straightforward and can be easily implemented

in Microsoft Excel for not only completed projects but also ongoing ones.

Based on the obtained results of Weibull analysis on the basis of $CR(t)$ data, the author therefore conclude that the proposed approach can provides a robust and effective method for managers to evaluate and compare overall performance of projects, and can be applicable in evaluating the project whose value has been determined.

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運用 Weibull分析於實獲值度量比較專案整體績效的統計分析

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摘要

大多數組織評估專案績效主要是以實獲值管理衡量專案成本和時程績效。然而，至目前專案管理文獻中僅提出少許工具以比較專案間整體績效。本研究介紹一簡單指標，CR(t) (以時間計關鍵比)，作為衡量專案的整體績效。此指標可依從專案成本系統中正常獲得的實獲值資料簡單地計算求得。根據此指標，本論文提出運用 Weibull分析實獲值度量以比較專案整體績效的統計分析，統計分析可在MS EXCEL上執行。本論文藉由五個案例專案依詳細步驟分析並加以比較專案間之績效。最後，依照分析比較結果，本論文所提方法可以提供管理者一個穩健且有效地評估和比較專案間之績效的方法，也可以適用於評估專案價值已確定的合約專案。

關鍵詞：實獲值度量;以時間計關鍵比;專案績效;Weibull分析

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