Introduction to Error Control Coding

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1. What Error Control Coding Is For

- In **data communications**, coding is used for control – ling transmission errors induced by channel noise or other impairments, such as fading and jamming, so that error-free communication can be achieved.
- In **data storage**, coding is used for controlling storage errors caused by storage medium defects, dust particles, and radiation so that error-free storage can be achieved.
- The block diagram of a typical data communication (or storage) system is shown in Figure 1.1.

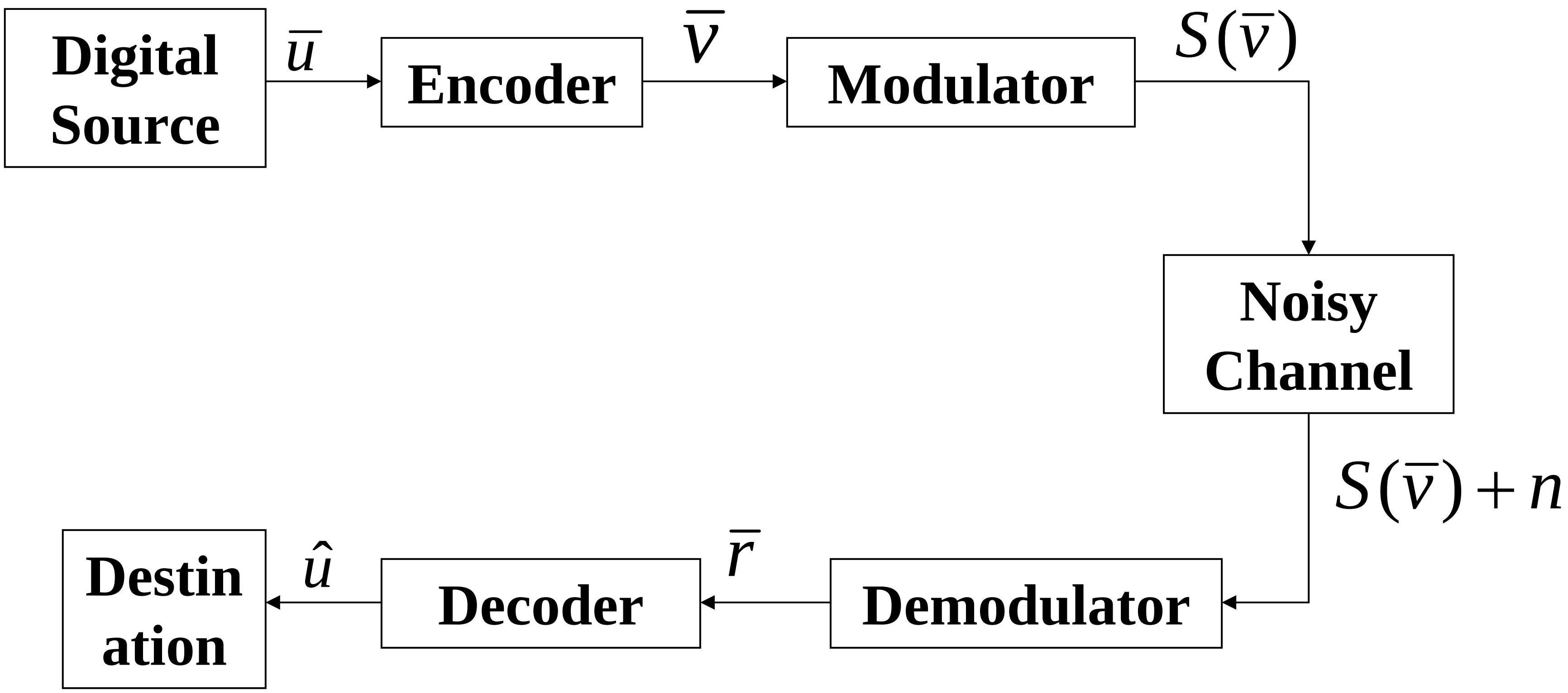


Fig. 1.1 The basic structure of a coded communication system

2. How Coding Can Be Achieved

- Coding is achieved by adding properly designed **redundant digits** to each message. These redundant digits are used for detecting or correcting transmission (or storage) errors.

3. Types of Coding

- **Block** and **convolutional** codings.
- **Block coding**: A message of k digits (usually bits) is mapped into a structured sequence of n digits (or bits), called a codeword. The mapping operation is called **encoding**. Each encoding operation is independent of the past encodings. That is, the encoder has no memory of history of past encodings. The collection of all codewords is called a block code.

- In general, both message and code symbols symbols are binary symbols, 0 and 1. In this case there are 2^k distinct messages. Corresponding to these 2^k distinct messages, these are 2^k binary codewords.
- The parameters, k and n, are called the message and code lengths respectively. In general, $n > k$.

- The rations, $\mathcal{R} = k / n$ and $\eta = (n - k) / n$, are called **code rate** and **redundancy,** respectively.
- $n - k$ **redundant digits** are added to each message for protection against errors.
- Example 1-1: Let $k = 3$ and $n = 6$. The following table gives a block code of length 6. The code rate is $\mathcal{R} = 1/2$.

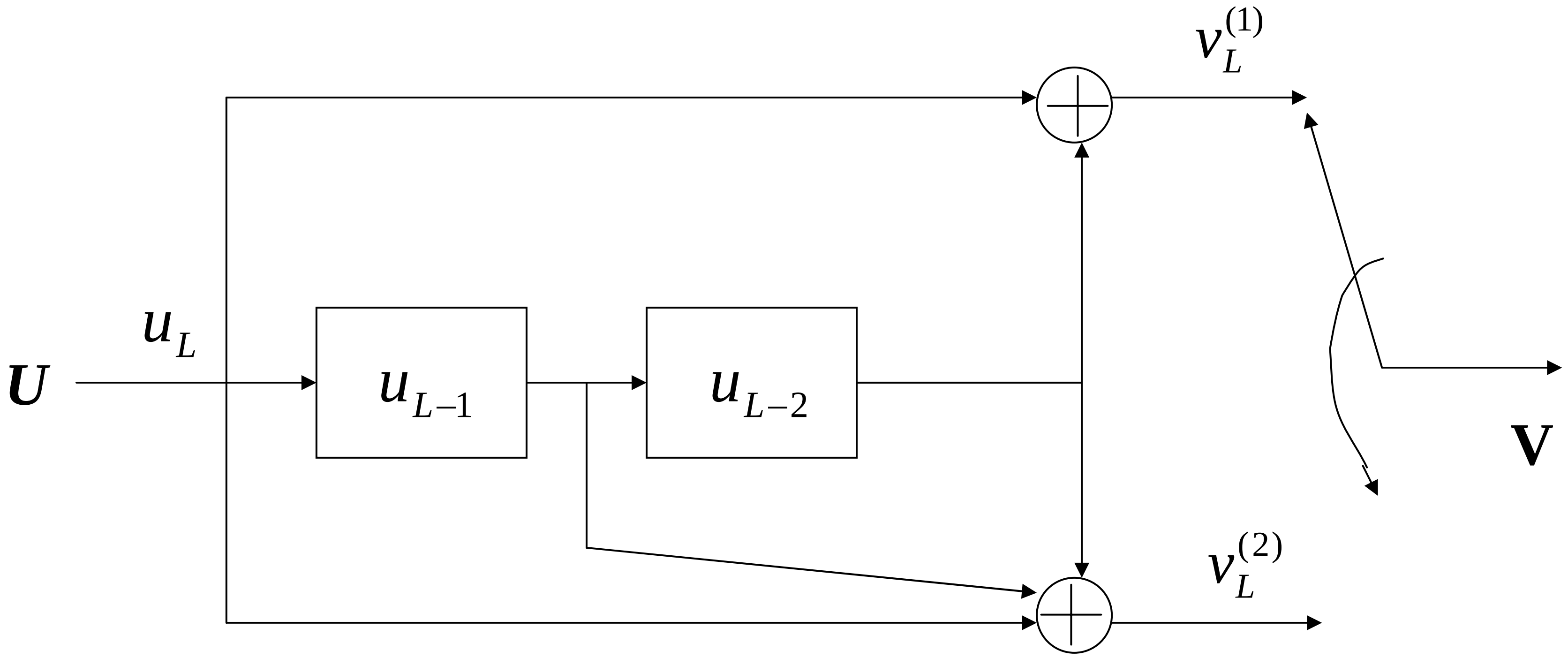
Example 1.1

message	codeword
(000)	(000000)
(100)	(011100)
(010)	(101010)
(110)	(110110)
(001)	(110001)
(101)	(101101)
(011)	(011011)
(111)	(000111)

- **Convolutional coding**: An information sequence is divided into (short) blocks of k digits each. Each k digit message is into an n digit coded block. The n digit coded block depends not only on the corresponding k digit message block but also on m ($\geqq 1$) **previous message** blocks. That is, the encoder has memory of order m.
- The encoder has k inputs and n outputs.

- An information sequence is encoded into a code sequence. The collection of all possible code sequences is called an (n, k, m) convolutional code.
- The parameters, k and n, are normally small, say $1 < k \leqq 8$ and $2 \leqq n \leqq 9$. Again, $k < n$ and the ratio $\mathcal{R} = k/n$ is called the code rate.
- Example 1-2: Let $k = 1$, $n = 2$ and $m = 2$. The following circuit generates a (2, 1, 2) convolutional code.

Example 1.2: Encoder for a (2,1,2) convolutional code



4. Types of Errors & Channels

- **Types of Errors:** Random and burst errors.
- **Types of Channels**

  (a) **Random error channels:** deep space channels, many satellite channels, line-of-sight transmission facilities, etc.

  (b) **Burst error channels:** radio links, terrestrial microwave links, wire and cable transmission.

5. Types of Codes

- Classification based on structure
  - (a) **block codes** – linear coders, cyclic codes
  - (b) **convolutional** code
- Classification based on the types of errors which they correct
  - (a) **random-error-correcting** codes
  - (b) **burst-error-correcting** codes
- Classification based on the types of code symbols
  - (a) error-correction codes
  - (b) error-detection codes

6. Typed of Error Control Schemes

- Forward-error-correction (FEC): An error correction code is used. After error correction, the decoded message is delivered to the user.
- Automatic repeat request (ARQ): An error detection code is used. If the presence of error is detected in a received word (or sequence), a retransmission is requested. The request signal is sent to the transmitter though a feedback channel. Retransmission continues until no errors being detected.
- Hybrid ARQ: A proper combination of FEC and ARQ

7. FEC vs ARQ

- ARQ:

  Types of ARQs

  (a) Stop-and-wait

  (b) Go-back-N

  (c) Selective-repeat

  Advantage: simple, easy to achieve high reliability.

  Disadvantage: feedback channel is needed, variable throughput.

- FEC:

    Advantage: no feedback channel is needed, constant throughput.

    Disadvantage: complex, hard to achieve high reliability.

8. Decoding

- Suppose a codeword corresponding to a certain message is transmitted over a noise channel.
- The receiver (or decoder), based on the encoding rules, and the noise characteristics of the channel, makes a decision which message was actually transmitted.
- This decision making operation is called **decoding.**
- The device which performs the decoding operation is called a **decoder**.
- There are two types of decoding based on the decisions mad by the decoder.

9. Hard and Soft Decision Decodings

- **Hard-Decision**: when binary coding is used, the modulator has only binary inputs (M = 2). If binary demodulator output quantization is used (Q = 2), the decoder has only binary inputs. In this case, the demodulator is said to make hard decisions. Decoding based on hard decisions made by the demodulator is called **hard decision decoding**.
- **Hard-decision decoding** is much easier to implement than soft-decision decoding. However, **soft-decision decoding** offers significant performance improvement over hard-decision decoding.

- **Soft-Decision**: If the output of demodulator consists of more than two quantization levels ($Q > 2$) or is left unquantized, the demodulator is said to make soft decisions. Decoding based on soft decision made by the demodulator is called **soft-decision decoding**.

10. Some channel models

- In a binary coded digital communication system, if the channel is an additive white Gaussian noise (AWGN) channel, hard decision made by the demodulator results in a binary symmetric channel (BSC) as shown in Figure 1.2. This channel is a memoryless channel.

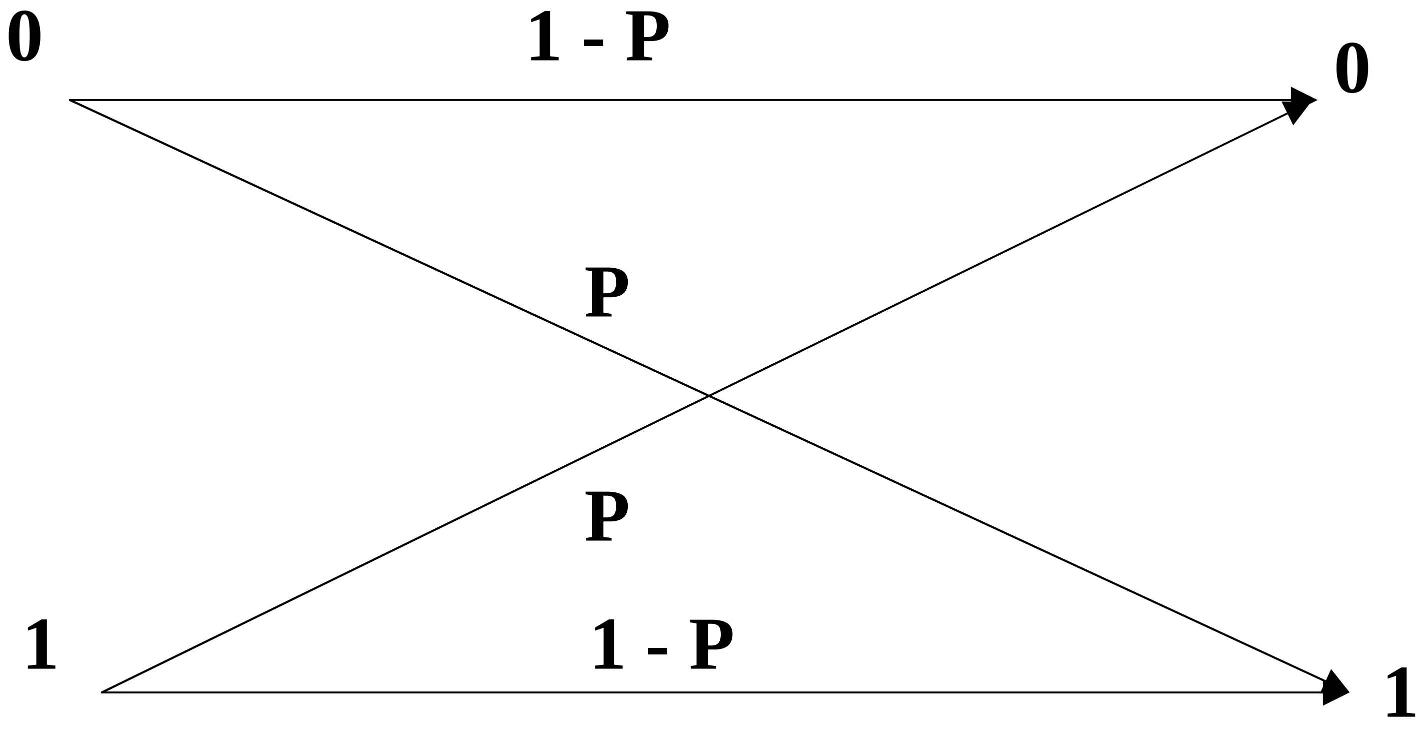


Figure 1.2 A binary symmetric channel

- Suppose the demodulator makes soft decision and has 8 output quantization levels (Q = 8). This we have a binary-input, 8-ary output discrete channel as shown in Figure 1.3. The 8-level quantization scheme is most frequently used in the soft-decision decoding systems.

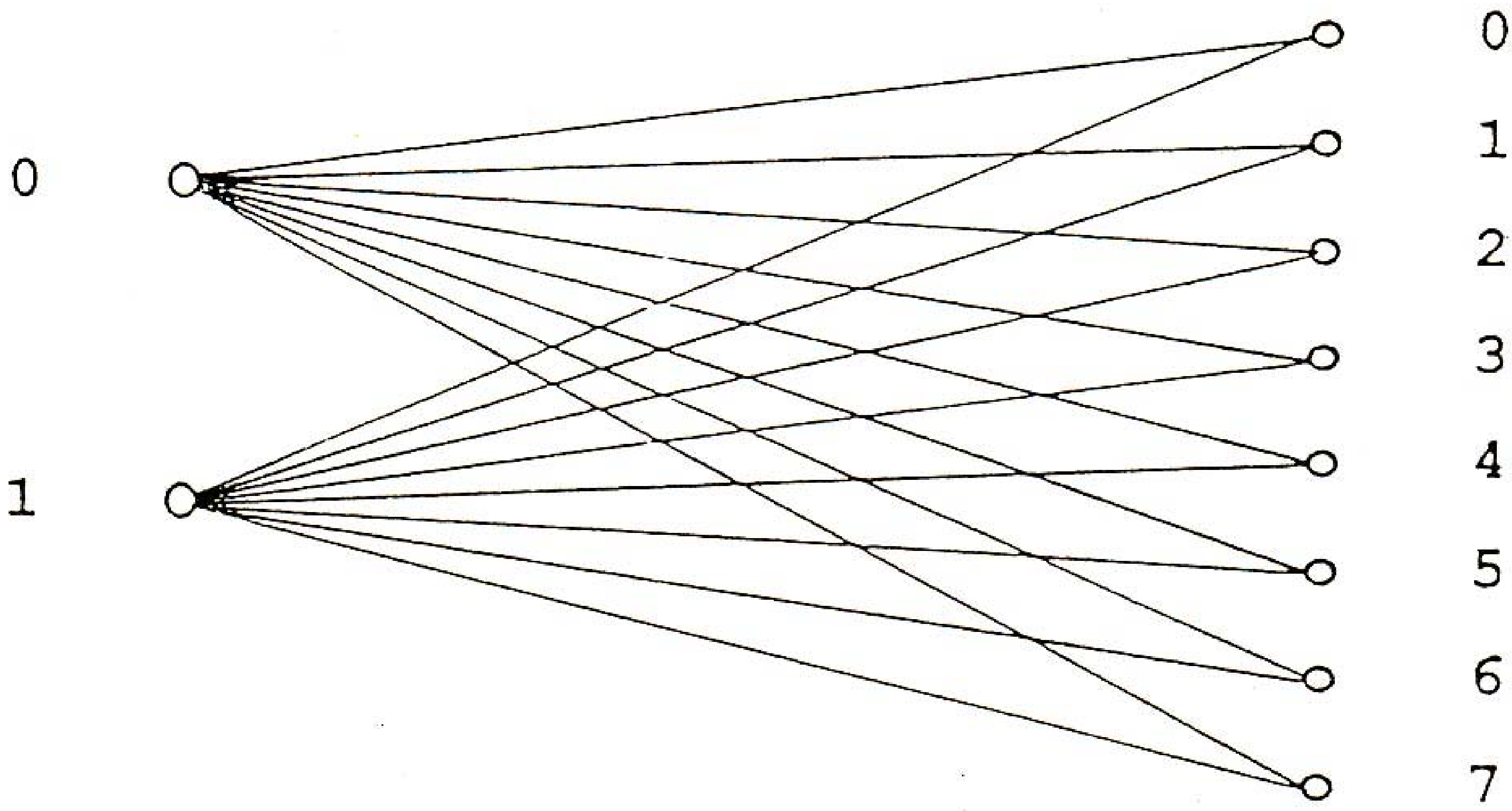


Figure 1.3 A binary-input, 8-ary output discrete channel

11. Maximum Likelihood Decoding

Optimum Decoding

- Suppose the codeword $\bar{v}$ corresponding to a certain message $\bar{u}$ is transmitted
- Let $\bar{r}$ be the corresponding output of the demodulator.
- The decoder must produce an estimate $\hat{u}$ of the message based on $\bar{r}$.

- Obviously, we would like to devise a decoding rule such that the probability of a decoding error is minimized, i.e,

$$\min \; P(\hat{u} \neq u)$$

- Such a decoding rule is called an optimum decoding rule.

Maximum Likelihood Decoding

- Suppose all the messages are **equally likely**. An optimum decoding can be done as follows.

  (1) The codeword $\bar{v}_j$ with largest conditional probability $p(\bar{r} \mid \bar{v}_j)$ is chosen as the estimate for the transmitted codeword

  (2) Then decode $\bar{v}_j$ into an estimate $\hat{u}$ for the transmitted message $\bar{u}$ based on the encoding rule, this decoding rule is called the **maximum likelihood decoding (MLD)**.

12. MLD for a BSC

- Let $\overline{a} = (a_1, a_2, \cdots, a_n)$ and $\overline{b} = (b_1, b_2, \cdots, b_n)$ be two binary sequences of *n* components. The Hamming distance between $\overline{a}$ and $\overline{b}$, denoted d($\overline{a}$, $\overline{b}$), is defined as the number of places where $\overline{a}$ and $\overline{b}$ differ.
- For example, let $\overline{a}$ = (1011011) and $\overline{b}$ = (0110101). Then d($\overline{a}$, $\overline{b}$) = 5.

- In coding for a BSC , every codeword and every received sequence are binary sequences.
- Suppose some codeword is transmitted and the received sequence is $\bar{r} = (r_1, r_2, \cdots r_n)$.
- For a codeword $\bar{v}_j$, the conditional probability is

$$p(\bar{r} \mid \bar{v}_j) = P^{d(\bar{r}, \bar{v}_j)} (1 - p)^{n - d(\bar{r}, \bar{v}_j)}$$

- For $p < ½$, $p\left(\bar{r} \mid \bar{v}_j\right)$ is a monotonically decreasing function of $d(\bar{r}, \bar{v}_j)$
- Then

$$p(\bar{r} \mid \bar{v}_j) > p(\bar{r} \mid \bar{v}_k)$$

if and only if $d(\bar{r}, \bar{v}_j) < d(\bar{r}, \bar{v}_k)$

- MLD :

(1) Compute $p\left(\bar{r} \mid \bar{v}_j\right)$ for all $d(\bar{r}, \bar{v}_j)$

(2) $\bar{v}_i$ is taken as the transmitted codeword if

for $d(\bar{r} \mid \bar{v}_i) < d(\bar{r} \mid \bar{v}_k)$

(3) Decode $\bar{v}_i$ into message $\hat{u}_i$

(4) The received vector $\bar{v}_i$ is decoded into the closest codeword.

- This also called the **minimum distance (nearest neighbor) decoding**.

13. Coding and Modulation Demodulation

- In most of coded digital communication systems, coding is designed and performed separately from modulation demodulation.
- Error control is provided by transmitting additional redundant bits in the code, which has the effect of **lowering** the **information bit rate** per channel bandwidth.
- In this case, **bandwidth efficiency** is traded for increased **power efficiency**.

- This is suitable for power-limited systems.
- However, when bandwidth efficiency is a major concern (such as in bandwidth-limited systems), the most effective method for error control is to combine coding and modulation as a single entity.
- In such an approach, coding is redefined as a process of imposing certain patterns on the transmitted signal.
- This definition obviously includes the traditional idea of redundancy.

- The combined modulation coding is called **coded modulation**.
- This error control technique is most suitable for **bandwidth-limited systems** where the available bandwidth must utilized effectively .